Zbirka nalog za srednje šole: MATEMATIKA

J. Dolenšek, M. Prosen, M. Vagaja: KOTNE FUNKCIJE. TRIGONOMETRIJA

Poglavje VIII.: Reševanje enačb

Str. 35. Naloge:

3 c) $\cos x = 0$

4 b) $\cos 2x = \frac{\sqrt{3}}{2}$

4 č) $\cos \frac{x}{3} = 0{,}834$

5 b) $\cos\left(\frac{x}{2} + \pi\right) = -1$

5 c) $\cos\left(2x + \frac{\pi}{2}\right) = -0{,}423$

6 č) $1 - 4\cos^2 x = 0$

6 d) $\sin^2 x - \cos^2 x = \frac{1}{2}$

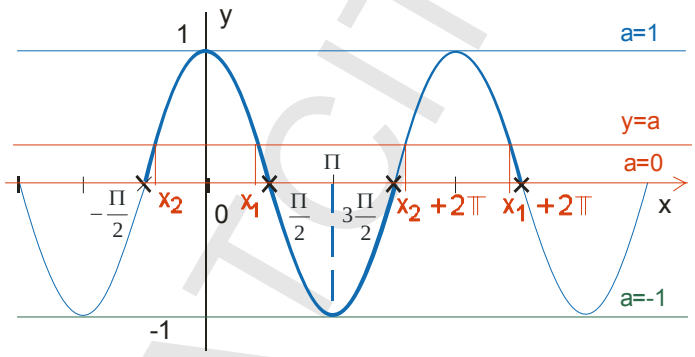
7 b) $4\sin^2 x - 3\cos^2 x = 1$

Razlaga:

Rešiti moramo enačbo v obliki $\cos x = a$, kjer je $-1 \le a \le 1$.

Posebej pogledam naslednje vrednosti cosinusa:

(glej tudi nalogo iz istega učbenika, poglavje IV.: Gafa funkcij sinus in kosinus, naloga 19 m).



$x_2 = -x_1$ oz. $x_1 + x_2 = 0$

(1) Ničle: $a = 0$ $\cos x = 0$

$x = \frac{\pi}{2} + k\pi$ $k \in \mathbb{Z}$ $\mathbb{Z} = \{0, \pm 1, \pm 2, ...\}$

(2) maksimume: $a=-1$ $\cos x=1$

$x=0+2k\pi \quad k\in\mathbb{Z}$

(3) minimume: $a=-1$ $\cos x=-1$

$x=\pi+2k\pi$

(4) za poljubno vrednost $-1\le a\le 1$: $\cos x=a$

$x_{1,2}=\pm\arccos a+2k\pi$

V splošnem velja formula (4), vendar za $a=0$, $a=-1$, $a=1$ raje računam po (1), (2), (3).

Rešitve

3 c) $\cos x=0$ (1)

$x=\frac{\pi}{2}+k\pi \quad k\in\mathbb{Z}$

4 b) $\cos 2x=\frac{\sqrt{3}}{2}$ (4)

$(2x)_{1,2}=\pm\arccos\frac{\sqrt{3}}{2}+2k\pi \quad k\in\mathbb{Z}$

$\cos 30^0=\frac{\sqrt{3}}{2}$

$(2x)_{1,2}=\pm\frac{\pi}{6}+2k\pi/:2$

$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$x_{1,2}=\pm\frac{\pi}{12}+k\pi$

4 č) $\cos\frac{x}{3}=0{,}834$ (4)

$\left(\frac{x}{3}\right)_{1,2}=\pm\arccos 0{,}834+2k\pi \quad k\in\mathbb{Z}$

$x_{1,2}=\pm 3\arccos 0{,}834+6k\pi$

5 b) $\cos\left(\frac{x}{2}+\pi\right)=-1$

$$\frac{x}{2}+\pi=\pi+2k\pi\,/.2$$
$$x+2\pi=2\pi+4k\pi$$
$$x=4k\pi \qquad k\in\mathbb{Z}$$

5 c) $\cos\left(2x+\frac{\pi}{2}\right)=-0{,}423$

$$\left(2x+\frac{\pi}{2}\right)_{1,2}=\pm\arccos(-0{,}423)+2k\pi \qquad k\in\mathbb{Z}$$

$$2x+\frac{\pi}{2}=+\arccos(-0{,}423)+2k\pi\,/.2$$
$$4x+\pi=-2\arccos(-0{,}423)+4k\pi$$
$$4x=2\arccos(-0{,}423)-\pi+4k\pi\,/:4$$
$$x_1=\frac{\arccos(-0{,}423)}{2}-\frac{\pi}{4}+k\pi$$

$$2x+\frac{\pi}{2}=-\arccos(-0{,}423)+2k\pi\,/.2$$
$$4x+\pi=-2\arccos(-0{,}423)+4k\pi$$
$$4x=-2\arccos(-0{,}423)-\pi+4k\pi\,/:4$$
$$x_2=\frac{-\arccos(-0{,}423)}{2}-\frac{\pi}{4}+k\pi$$

6 č) $1-4\cos^2 x=0$

Preoblikujem v obliko za uporabo (4):

$$-4\cos^2 x=-1$$
$$\cos^2 x=\frac{1}{4}$$
$$\cos x=\pm\frac{1}{2}$$

$\cos x=\frac{1}{2}$ (4)

$$x_{1,2}=\pm\arccos\frac{1}{2}+2k\pi \qquad k\in\mathbb{Z}$$
$$x_{1,2}=\pm\frac{\pi}{3}+2k\pi$$

$\cos x=-\frac{1}{2}$ (4)

$$x_{3,4}=\pm\arccos\left(-\frac{1}{2}\right)+2k\pi$$
$$x_{3,4}=\pm\frac{2\pi}{3}+2k\pi$$

6 d) $\sin^2 x - \cos^2 x = \frac{1}{2}$

Uporabim formulo:

$\sin^2 x + \cos^2 x = 1$

in

$\sin^2 x = 1 - \cos^2 x$

$$1 - \cos^2 x - \cos^2 x = \frac{1}{2}$$

$$1 - 2\cos^2 x = \frac{1}{2} / .2$$

$$2 - 4\cos^2 x = 1$$

$$-4\cos^2 x = -1$$

$$\cos^2 x = \frac{1}{4}$$

Za nadaljevanje glej nalogo 6 č)

7 b) $4\sin^2 x - 3\cos^2 x = 1$

$$4(1 - \cos^2 x) - 3\cos^2 x = 1$$

$$4 - 4\cos^2 x - 3\cos^2 x = 1$$

$$4 - 7\cos^2 x = 1$$

$$-7\cos^2 x = -3$$

$$\cos^2 x = \frac{3}{7}$$

$$\cos x = \pm\sqrt{\frac{3}{7}}$$

$\cos x = \sqrt{\frac{3}{7}}$

$x_{1,2} = \pm\arccos\sqrt{\frac{3}{7}} + 2k\pi \quad k \in \mathbb{Z}$

$\cos x = -\sqrt{\frac{3}{7}}$

$x_{3,4} = \pm\arccos\left(\frac{\sqrt{3}}{7}\right) + 3k\pi \quad k \in \mathbb{Z}$